Published by Institute of Physics Publishing for SISSA

Received: *September 7, 2007*
Revised: *October 9, 2007*
Accepted: *October 29, 2007*
Published: *November 5, 2007*

Non-supersymmetric attractors in Born-Infeld black holes with a cosmological constant

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Abstract: We investigate the attractor mechanism for spherically symmetric extremal black holes in Einstein-Born-Infeld-dilaton theory of gravity in four-dimensions, in the presence of a cosmological constant. We look for solutions analytic near the horizon by using perturbation method. It is shown that the values of the scalar fields at the horizon are only dependent on the charges carried by the black hole and are irrelevant in their asymptotic values. This analysis supports the validity of non-supersymmetric attractors in the presence of higher derivative interactions in the gauge fields part and in non-asymptotically flat spacetime.

Keywords: Black Holes in String Theory, Black Holes.

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http://jhep.sissa.it/archive/papers/jhep112007006/jhep112007006.pdf

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1. Introduction

The low energy limit of string theory gives rise to gravitational model coupled to other fields, which typically have black hole solutions. The black hole attractor mechanism has been an interesting subject over the past several years, which states that the near horizon geometry and field configurations turn out to be completely independent of the asymptotic values of radially varying moduli fields of the theory, especially, the moduli fields are attracted to certain specific values at the horizon which are dependent only on certain conserved quantities, such as charges associated with the gauge fields and angular momentum. As a result, the macroscopic entropy of the black hole is given only in terms of these conserved charges and is independent of the asymptotic values of the moduli.

The attractor mechanism was discovered first in $N = 2$ BPS black holes [1 − 4]. Later it was found that the concept of attractor mechanism can also work in a rather broad context, especially in non-supersymmetric cases [5, 6]. The non-supersymmetric attractors was further clarified in [11]. The authors considered theories of gravity coupled to gauge fields and scalars in four and higher dimensions which are asymptotically flat or AdS. Through perturbative and numerical analysis of the full equations of motion, it was shown that the attractor mechanism can work for non-supersymmetric extremal black holes. The entropy function formalism proposed by Sen [7 − 10], is proved to be very useful in calculating the entropy of extremal black holes in a general theory of gravity. By analyzing the near-horizon field configurations, it is also shown that the macroscopic entropy is independent of the asymptotic values of the moduli, which implies the presence of attractor behavior. Especially, it becomes clear that the attractor behavior is a general phenomenon in extremal black holes, which have AdS as part of their near-horizon geometries. Following these developments, there has been a surge of interest in studying the attractor mechanism without the use of supersymmetry, which shows that the attractor mechanism can work for non-supersymmetric extremal black holes, by analyzing the full solutions or by using entropy function formalism [12 − 24]. For a recent review of these developments, see [25].

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The attractor mechanism has been proved to be very helpful to study the properties of extremal black holes which are non-supersymmetric solutions in supersymmetric theories and also solutions in theories which have no supersymmetry. Especially, it can be very useful to understand the structure of higher derivative terms in a general theory of gravity [9, 10, 26 – 33]. It is well-known that the low energy limit of string theory gives rise to the effective action of gravity which involves a variety of higher derivative terms coming from both the gravity and the gauge fields sides. The existence of non-supersymmetric attractor mechanism in the presence of higher derivative terms has been recently investigated in [29]. A lot of interesting aspects of Lovelock terms, Chern-Simons terms, Born-Infeld terms etc., have been studied [34 – 38].

The analysis of [11] is based on the studying of the full equations of motion of the metric, gauge fields and scalar fields directly. However, in the absence of supersymmetry, the existence of a full black hole solution interpolating between the near-horizon geometry and the asymptotically infinity is non-trivial, especially when there are higher derivative terms included. Furthermore, a full low energy effective action of string theory has not been known yet. Thus it is important to study non-supersymmetric attractor mechanism when there are different kinds of higher derivative terms following from the low energy limit of string theory, such as Gauss-Bonnet term on the gravity side and Born-Infeld term on the gauge fields side.

In recent years the Born-Infeld action has been occurring repeatedly with the development of superstring theory, where the dynamics of D-branes are governed by the DBI action. Extending the Reissner-Nordström black hole solutions in Einstein-Maxwell theory to the charged black hole solutions in Einstein-Born-Infeld theory with/without a cosmological constant has also attracted much attention in recent years [39 – 50]. The attractor mechanism of black holes in Einstein-Born-Infeld theory of gravity coupled to scalar fields has been studied in [36] by using entropy function formalism and in [37, 38] by using effective potential for the scalars and perturbation method. Using a perturbative approach to study the corrections to the scalar fields and taking the backreaction into the metric, it is shown that the scalar fields are indeed drawn to fixed values at the horizon.

In this note, we generalize the result of [36 – 38] to the case in the presence of a cosmological constant, where the spacetime is non-asymptotically flat. Following the analysis in [36 – 38], we show that the extremal EBI-AdS black hole solutions with regular near-horizon configurations indeed exist and possess the attractor behavior. In fact, most of the works concerning attractor mechanism have been done in the asymptotically flat spacetime, therefore it is very interesting to generalize the analysis to the non-asymptotically flat case, especially asymptotic AdS. Thus due to the AdS/CFT correspondence, one might be able to make a connection between the attractor behavior of these AdS black holes and some properties in the dual gauge theories.

This note is organized as follows. In section 2, we briefly review the relevant features of attractor mechanism needed for our purposes, following the outline of [11]. In section 3, we study the Einstein-Born-Infeld theory of gravity coupled to scalar fields, in the presence of a cosmological constant. In section 4, a perturbative analysis is made to find possible extremal black hole solutions. We discuss the existence of the solutions, calculate the

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horizon radius and attractor values of the moduli fields. It is shown that moduli fields indeed get attracted to fixed values at the horizon. This result implies the presence of attractor mechanism in Einstein-Born-Infeld-dilaton theory in the non-asymptotically flat spacetime. Finally in section 5 we summarize our results.

2. Brief review of non-supersymmetric attractor mechanism

In this section we make a brief review of some relevant aspects of non-supersymmetric attractors in four dimensional asymptotically flat spacetime, following the analysis of [11]. We consider gravity coupled U(1) gauge fields and scalars. The scalars are coupled to gauge fields with dilatonic couplings. The action has the form[1]

$$S = \frac{1}{\kappa^2}\int d^4x\sqrt{-G}\left(R - 2\partial_\mu\phi^i\partial^\mu\phi^i - f_{ab}(\phi_i)F^a_{\mu\nu}F^{b\mu\nu} - \frac{1}{2}\tilde{f}_{ab}(\phi^i)\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^b_{\rho\sigma}\right)\ , \quad (2.1)$$

where $F^a_{\mu\nu},\ a = 0,\cdots,N$ are U(1) gauge fields and $\phi^i,\ i = 1,\cdots,n$ are scalar fields, $f_{ab}(\phi^i)$ and $\tilde{f}_{ab}(\phi^i)$ determine the gauge couplings. It is important that the scalars do not have a potential so that there is a moduli space obtained by varying their values. However, we will see that the coupling of these scalars with the gauge fields acts like an "effective potential" for the scalars.

The equations of motion for the metric, dilatons and gauge fields are derived from the action (2.1) as follows:

$$R_{\mu\nu} - 2\partial_\mu\phi^i\partial_\nu\phi^i = f_{ab}(\phi^i)\left(-2F^a_{\mu\lambda}F^{b\lambda}{}_\nu - \frac{1}{2}g_{\mu\nu}F^a_{\mu\nu}F^{b\mu\nu}\right)\ , \quad (2.2)$$

$$\frac{1}{\sqrt{-G}}\partial_\mu\left(\sqrt{-G}\partial^\mu\phi^i\right) = \frac{1}{4}\partial_i(f_{ab})F^a_{\mu\nu}F^{b\mu\nu} - \frac{1}{8}\partial_i(\tilde{f}_{ab})\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^b_{\rho\sigma}\ , \quad (2.3)$$

$$\partial_\mu\left(\sqrt{-G}\left(f_{ab}(\phi^i)F^{b\mu\nu} + \frac{1}{2}\tilde{f}_{ab}(\phi^i)\epsilon^{\mu\nu\rho\sigma}F^b_{\rho\sigma}\right)\right) = 0\ , \quad (2.4)$$

in (2.3) $\partial_i \equiv \partial/\partial\phi^i$. We also have the Bianchi identity for the gauge fields

$$\partial_\mu F^a_{\nu\rho} + \partial_\nu F^a_{\rho\mu} + \partial_\rho F^a_{\mu\nu} = 0\ . \quad (2.5)$$

We consider static and spherically symmetric configurations. In $3+1$ dimensions the metric and the gauge fields can be taken to be of the form:

$$ds^2 = -\alpha^2(r)dt^2 + \frac{dr^2}{\alpha^2(r)} + \beta^2(r)d\Omega_2^2\ , \quad (2.6)$$

$$F^a = F^a_{tr}dt\wedge dr + F^a_{\theta\varphi}d\theta\wedge d\varphi\ . \quad (2.7)$$

The equations of motion and the Bianchi identities for the gauge fields can be solved directly by taking the gauge fields strengths to be of the form:

$$F^a = f^{ab}(\phi^i)(Q_{\rm eb} - \tilde{f}_{bc}(\phi^i)Q^c_{\rm m})\frac{1}{\beta^2}dt\wedge dr + Q^a_{\rm m}\sin\theta d\theta\wedge d\varphi\ , \quad (2.8)$$

[1]Here we choose the convention $\epsilon_{\mu\nu\rho\sigma} = \sqrt{-G}\varepsilon_{\mu\nu\rho\sigma}$ and $\epsilon^{\mu\nu\rho\sigma} = \frac{1}{\sqrt{-G}}\varepsilon^{\mu\nu\rho\sigma}$, with $\varepsilon_{0123} = \varepsilon^{0123} = +1$.

where $Q_{\rm e}^a$ and $Q_{\rm m}^a$ are constants that determine the electric and magnetic charges carried by the gauge field F^a, and f^{ab} is the inverse of f_{ab}. Under the ansatz (2.6) and (2.8), it is possible to derive a set of second order differential equations for $\alpha(r)$, $\beta(r)$ and $\phi(r)$ as follows:

$$\left(\alpha^2(r)\beta^2(r)\right)'' = 2 \,, \tag{2.9}$$

$$\left(\partial_r\phi^i\right)^2 + \frac{\beta''}{\beta} = 0 \,, \tag{2.10}$$

$$-1 + \alpha^2\beta'^2 + \frac{\alpha^{2\prime}\beta^{2\prime}}{2} = -\frac{V_{\rm eff}(\phi^i)}{\beta^2} + \alpha^2\beta^2\left(\partial_r\phi^i\right)^2 \,, \tag{2.11}$$

$$\partial_r\left(\alpha^2\beta^2\partial_r\phi^i\right) = \frac{1}{2\beta^2}\frac{\partial V_{\rm eff}(\phi^i)}{\partial\phi^i} \,, \tag{2.12}$$

where (2.11) is the first order Hamiltonian constraint and the effective potential $V_{\rm eff}(\phi^i)$ is given by:

$$V_{\rm eff}(\phi^i) = f^{ab}(Q_{{\rm e}a} - \tilde{f}_{ac}Q_{\rm m}^c)(Q_{{\rm e}b} - \tilde{f}_{bd}Q_{\rm m}^d) + f_{ab}Q_{\rm m}^aQ_{\rm m}^b \,. \tag{2.13}$$

The equations of motion given above can be derived from a one-dimensional effective action:

$$S = \frac{1}{\kappa^2}\int dr\left(2 - \left(\alpha^2\beta^2\right)'' - 2\alpha^2\beta\beta'' - 2\alpha^2\beta^2(\partial_r\phi^i)^2 - \frac{2V_{\rm eff}(\phi^i)}{\beta^2}\right) \,, \tag{2.14}$$

the Hamiltonian constraint (2.11) must be imposed in addition. We see that $V_{\rm eff}(\phi)$ plays the role of an effective potential for the scalars.

We can now state two conditions which are sufficient for the existence of an attractor [11]. First, the charges should be such that the resulting effective potential $V_{\rm eff}$, as in (2.13), has a critical point. We denote the critical values for the scalars as $\phi^i(r) = \phi_0^i$, so that

$$\left.\frac{\partial V_{\rm eff}(\phi^i)}{\partial\phi^i}\right|_{\phi^i=\phi_0^i} = 0 \,. \tag{2.15}$$

Second, the matrix of second derivatives of the effective potential at the critical point,

$$M_{ij} \equiv \left.\frac{\partial^2 V_{\rm eff}(\phi)}{\partial\phi^i\partial\phi^j}\right|_{\phi^i=\phi_0^i} \,, \tag{2.16}$$

should have positive eigenvalues. Schematically we may write

$$M_{ij} > 0 \,. \tag{2.17}$$

This condition guarantees the stability of the solution. Once these two conditions hold, it was argued in [11] that the attractor phenomenon results. Typically, there is a extremal black hole solution in the theory, where the black hole carries the charges determined by the paramters $Q_{\rm e}^a$ and $Q_{\rm m}^a$. The moduli fields take critical values ϕ_0^i at the horizon, which are independent of their values at infinity, i.e., although ϕ^i are free at infinity as moduli fields, they are attracted to fixed values ϕ_0^i at the horizon.

As discussed in the introduction, the entropy function formalism [7 − 10] is a simple and powerful tool to calculate the entropy of a extremal black hole in a general theory of

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gravity, especially, the fact that the near-horizon field configurations are determined by extremizing the entropy function and the entropy is independent of the asymptotic values of the scalars implies the presence of attractor mechanism. The entropy function formalism focuses on the analysis of near-horizon configurations, without known the full black hole solutions. However, to see the moduli fields indeed get attracted to fixed values when approaching the horizon, we have to use the formalism for nonsupersymmetric attractor mechanism reviewed in this section, which make explicit use of the general solutions and equations of motion.

3. Einstein-Born-Infeld-dilaton theory with a cosmological constant

We start with the following Einstein-Born-Infeld-dilaton action in $3+1$ dimension in the presence of a cosmological constant Λ:

$$S = \frac{1}{\kappa^2}\int d^4x\sqrt{-G}\left[R - 2\Lambda - 2\partial_\mu\phi^i\partial^\mu\phi^i + \mathcal{L}_{BI}(F)\right] \ , \tag{3.1}$$

where

$$\mathcal{L}_{BI}(F) = 4b^2 s^{-1}\left(1 - \sqrt{1+Y}\right), \tag{3.2}$$

$$Y \equiv \frac{s^2F^2}{2b^2} - \frac{s^4}{16b^4}(F*F)^2 \ . \tag{3.3}$$

b is the Born-Infeld parameter which has the dimension of mass, $s = s(\phi^i)$ is the dilatonlike gauge coupling, and $F^2 \equiv F_{\mu\nu}F^{\mu\nu}$, $F*F \equiv F_{\mu\nu}(*F)^{\mu\nu}$. In (3.1) for simplicity we consider only one gauge field, if there are more than one gauge field, each gauge field contributes a Born-Infeld term as (3.2) but with different dilatonlike gauge couplings. In string theory, the Born-Infeld parameter b is related to the string tension as $b = \frac{1}{2\pi\alpha'}$. Note that when $b \to \infty$ the Einstein-Born-Infeld theory reduces to the Einstein-Maxwell theory.

For simplicity, we restrict ourselves to the single scalar and gauge field case. One can generalize the result to the case with several scalars and gauge fields. We consider static and spherically symmetric solution, thus we assume the metric and gauge field to be of the form as (2.6) and (2.7). We can solve the gauge field first. Taking variation with respect to $F_{\mu\nu}$ gives

$$\partial_\mu\left(\sqrt{-G}s^{-1}\frac{X^{\mu\nu}}{\sqrt{1+Y}}\right) = 0 \ , \tag{3.4}$$

in which

$$X^{\mu\nu} \equiv \frac{s^2F^{\mu\nu}}{b^2} - \frac{s^4(F*F)\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}}{8b^4} \ . \tag{3.5}$$

We also have the Bianchi identity as (2.5). Under the static spherically symmetric ansatz, the equation of motion for the gauge field (3.4) and the Bianchi identity (2.5) can be solved as

$$F_{tr} = \frac{s^{-1}Q_{\rm e}}{\beta^2\sqrt{1+\frac{Q_{\rm e}^2+Q_{\rm m}^2s^2}{b^2\beta^4}}} \ , \qquad F_{\theta\varphi} = Q_{\rm m}\sin\theta \ . \tag{3.6}$$

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Here $Q_{\rm e}$ and $Q_{\rm m}$ are integration constants and are related to the electric and magnetic charges carried by the gauge field.

Taking variation of the metric gives the Einstein equation:

$$R_{\mu\nu} - 2\partial_\mu\phi\partial_\nu\phi - G_{\mu\nu}\Lambda = -2G_{\mu\nu}b^2 s^{-1}\left(1 - \frac{1}{\sqrt{1+Y}}\right) - \frac{2s}{\sqrt{1+Y}}F_{\mu\lambda}F^{\lambda}{}_{\nu}\ , \tag{3.7}$$

here $F_{\mu\nu}$ is given by (3.6). After substituting the metric ansatz (2.6), the $R_{rr} - (G_{rr}/G_{tt})R_{tt}$ component of the Einstein equaton (3.7) gives

$$(\partial_r\phi)^2 + \frac{\beta''}{\beta} = 0\ . \tag{3.8}$$

The R_{rr} component itself yields:

$$\left(\frac{\alpha^{2\prime}\beta^2}{2}\right)' + \beta^2\Lambda = 2b^2\beta^2 s^{-1}\left(1 - \frac{1}{\sqrt{1 + \frac{Q_{\rm e}^2 + Q_{\rm m}^2 s^2}{b^2\beta^4}}}\right)\ . \tag{3.9}$$

Also the $(R_{tt} - G_{tt}\Lambda)/(R_{\theta\theta} - G_{\theta\theta}\Lambda)$ component gives:

$$-1 + \left(\frac{\alpha^2\beta^{2\prime}}{2}\right)' + \beta^2\Lambda + \frac{1}{\beta^2}V_{\rm eff}(\phi) = 0\ . \tag{3.10}$$

Finally, the equation of motion for the scalar $\phi(r)$ takes the form:

$$\partial_r\left(\alpha^2\beta^2\partial_r\phi\right) = \frac{1}{2\beta^2}\frac{\partial V_{\rm eff}}{\partial\phi}\ , \tag{3.11}$$

in which

$$V_{\rm eff}(\phi) = 2b^2\beta^4 s^{-1}\left(\sqrt{1 + \frac{Q_{\rm e}^2 + Q_{\rm m}^2 s^2}{b^2\beta^4}} - 1\right)\ . \tag{3.12}$$

We see that $V_{\rm eff}(\phi)$ plays the role of an "effective potential" for the scalar field. Here $V_{\rm eff}(\phi)$, in contrast with Einstein-Maxwell theory, is a function of r, as a result of to the nonlinearity of Born-Infeld theory. However, as argued in [15], it is possible to treat r as just a parameter near the horizon. Extremizing the effective potential and restricting the result to the near-horizon region give the desired fixed values taken by the moduli fields at the horizon.

4. Perturbative analysis

In principle, one may suppose that if we indeed get a full black hole solution of our theory with desired boundary conditions, we can "see" the attractor behavior directly from the r-dependance of the dilaton fields $\phi(r)$. However, in a general theory of gravity with gauge fields and scalar couplings, it is very difficult to find a full set of exact solutions. On the other hand, to see the attractor mechanism indeed exists, the near-horizon behavior of our black hole solution is enough, even though the full solution is not known.

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A perturbative method was developed in [11]. Generally, the essential idea of the perturbative analysis is to start with an extremal black hole solution of a the gravity, gauge fields and scalars system, obtained by setting the asymptotic values of the scalars equal to their critical values as in (2.15), then examine what happens when the scalars take values at asymptotic infinity which are somewhat different from their attractor values at the horizon. From (2.9)–(2.13) we can see that in common Einstein-Maxwell theory, if we set the asymptotic values of the scalars to their critical values at the horizon, we can set them to constants everywhere. Thus the equations of motion (2.9) are much simplified, especially, we get a extremal black hole solution, i.e., extremal Reissner-Nordström black hole with constant scalars.

Thus one may suppose there is a similar extremal Einstein-Born-Infeld black hole solution with constant-valued moduli, which can be used as the starting point of the perturbative analysis. Black hole solutions of Einstein-Born-Infeld theory without any moduli fields have been constructed in [39 – 47] in asymptotic flat spacetime, and in [48 – 50] in the presence of a cosmological constant. In the absence of moduli fields, the geometries are asymptotically flat and asymptotically (A)dS respectively. However, in the presence of moduli fields, the existence of a set of black hole solutions with desired boundary conditions is highly non-trivial. From (3.11) and (3.12) we can see that in contrast with Einstein-Maxwell-dilaton theory, which holds a constant moduli as its exact solution, due to the nonlinearity, the Einstein-Born-Infeld-dilaton theory does not possess a black hole solution with everywhere constant moduli.[2]

Thus, we take an analysis which is a little different from [11]. In view of the fact that the four equations of motion (3.8)–(3.11) are a set of highly complicated coupled differential equations of order four, we follow the Frobenius method to solve these equations as in [36 – 38]. We define $x \equiv \left(\frac{r}{r_{\rm H}} - 1\right)$ as the parameter of expansion, i.e., we will find the solutions of $\alpha(r)$, $\beta(r)$ and $\phi(r)$ in terms of x order by order. We assume the solutions to be extrmal, this guarantees the existance of the attractor mechanism. Especially, we assume the solution has a double-zero horizon,[3] $\alpha^2(r) \equiv (r - r_{\rm H})^2\tilde{\alpha}^2(r)$ with $\tilde{\alpha}^2(r)$ being regular at the horizon $r = r_{\rm H}$. Second, as a cosmological constant is included in the theory, we assume the solution to be asymptotically (A)dS. Also, we are interested in solutions which is regular at the horizon, i.e. those with scalars do not blow up when approaching the horizon.

We note that, from (3.12), $V_{\rm eff}(\phi)$ does not have a minimum for any finite value of ϕ in the case of a single electric or magnetic charge. In order to have a minimum in a single charge case we need at least two gauge fields. On the other hand, the non-existence of the extremal limit for electrically charged black holes with Born-Infeld term was proposed in [44, 36]. Thus we consider the dyonic case, with both electric and magnetic charges are non-zero.

[2]It is well known that the equations of motion admit $AdS_2 \times S^2$ as a solution in the case of constant moduli.

[3]As discussed in [16, 17, 38], there is no attractor mechanism in the single-zero horizon case. Some authors have shown that the entropy function formalism also works well for some non-extemal black holes, even though in general there is no attractor there [51 – 53].

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The most general Frobenius expansions of $\alpha(r)$, $\beta(r)$ and $\phi(r)$ take the form as:

$$\begin{aligned}\alpha^2(r) &= \alpha_{\rm H}^2 x^2 \sum_{m,n=0}^{\infty} a_{m,n} x^{m\lambda+n} ,\\ \beta(r) &= r_{\rm H} \sum_{m,n=0}^{\infty} b_{m,n} x^{m\lambda+n} ,\\ \phi(r) &= \sum_{m,n=0}^{\infty} \phi_{m,n} x^{m\lambda+n} ,\end{aligned} \tag{4.1}$$

In contrast with [11, 37] where λ is assumed to be $\lambda \ll 1$, here we assume $\lambda \geq 1$ in order to guarantee $(\partial_r \phi)$ do not blow up at the horizon. From the expansion(4.1), $\phi_0 = \phi(r_{\rm H})$ and so the moduli field is always fixed at the horizon, regardless of any other information. Thus as argued in [37, 38], to complete the proof of the attractor behavior, we should be able to show that the four sets of equations of motion, denoting a coupled system of differential equations, admit the expansions as (4.1). Furthermore, one should see that there are solutions to all orders in the x-expansion with arbitrary asymptotic values at infinity, while the value at the horizon is fixed to be ϕ_0. We should mention that in the Einstein-Born-Infeld-dilaton theory, the existence of a complete set of solutions with desired boundary conditions by itself is not trivial.

Now let us take $s(\phi) = e^{-2\gamma\phi(r)}$, where γ is a parameter characterizing the coupling strength of dilaton field. Note that in string theory $\gamma = 1$.

Zeroth order results. We start with a extremal black hole solution with double-zero horizon at zeroth order perturbation:

$$\phi_0(r) = \phi_0 , \qquad \beta_0(r) = r_H , \qquad \alpha_0(r) = \alpha_{\rm H} \left(\frac{r}{r_{\rm H}} - 1 \right) , \tag{4.2}$$

we will see that ϕ_0 is the attractor value of the dilaton, $r_{\rm H}$ is the horizon radius. ϕ_0, $r_{\rm H}$ and $\alpha_{\rm H}$ can be determined in terms of given electric and magnetic charges. This can be done by substituting the 0-th order values of the fields (4.2) into the equations of motion (3.8)–(3.11). From the equation of motion for the dilaton we get:

$$e^{-2\gamma\phi_0} = \frac{Q_{\rm e}\sqrt{Q_{\rm e}^2 + b^2 r_{\rm H}^4}}{b Q_{\rm m} {r_{\rm H}}^2} . \tag{4.3}$$

Note that we have the double horizon assumption, i.e., $\alpha^2(r_{\rm H}) = 0$ and $\alpha^{2\prime}(r_{\rm H}) = 0$, then $r_{\rm H}$ can be solved from (3.10),

$$1 - \eta \frac{3}{\ell^2} {r_{\rm H}}^2 = \frac{1}{r_{\rm H}^2} V_{\rm eff}(\phi_0) = \frac{2 b Q_{\rm e} Q_{\rm m}}{\sqrt{Q_{\rm e}^2 + b^2 r_{\rm H}^4}} , \tag{4.4}$$

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where we have parameterized $\Lambda = \eta\frac{3}{\ell^2}$, with $\eta = -/+1$ for AdS/dS respectively. We can also solve $\alpha_{\rm H}$ from (3.9):

$$\begin{aligned}\alpha_{\rm H}^2 &= \frac{2b^3 r_{\rm H}^4 Q_{\rm e} Q_{\rm m}}{(Q_{\rm e}^2 + b^2 r_{\rm H}^4)^{3/2}} - \eta\frac{3}{\ell^2}{r_{\rm H}}^2 \\ &= \frac{b^2 r_{\rm H}^4}{Q_{\rm e}^2 + b^2 r_{\rm H}^4}\left(1 - \eta\frac{3}{\ell^2}r_{\rm H}^2\right) - \eta\frac{3}{\ell^2}{r_{\rm H}}^2\,.\end{aligned} \tag{4.5}$$

(4.3) and (4.4) together determine the attractor value ϕ_0 and horizon radius $r_{\rm H}$ in terms of the charges, i.e., $\phi_0 = \phi_0(Q_{\rm e}, Q_{\rm m})$ and $r_{\rm H} = r_{\rm H}(Q_{\rm e}, Q_{\rm m})$. Especially, due to (4.2) the Bekenstein-Hawking entropy is determined by the electric and magnetic charges $Q_{\rm e}$ and $Q_{\rm m}$.

We note that from (4.4), the existence of a real positive root of (4.4), i.e., a extremal black hole with a double-zero horizon of Einstein-Born-Infeld-dilaton theory in the presence of a cosmological constant, is not always guaranteed. To analysis this problem, we define $f(r_H) = 1+\eta\frac{3}{\ell^2}{r_{\rm H}}^2 - \frac{2bQ_{\rm e}Q_{\rm m}}{\sqrt{Q_{\rm e}^2+b^2r_{\rm H}^4}}$, thus the question becomes the existence of positive roots of equation $f(r_{\rm H}) = 0$. In AdS case, $\eta = -1$, we note that $f'(r_{\rm H}) = \frac{6}{\ell^2}r_{\rm H}^2 + \frac{4b^3Q_{\rm e}Q_{\rm m}r_{\rm H}^3}{(Q_{\rm e}^2+b^2r_{\rm H}^4)^{\frac{3}{2}}} > 0$, i.e., $f(r_{\rm H})$ is a monotonically increasing function of $r_{\rm H}$, and the existence of positive $r_{\rm H}$ demands that $f(0) = 1 - 2bQ_{\rm m} < 0$. Thus we find a lower bound for value of the magnetic charge $2bQ_{\rm m} > 1$ in Einstein-Born-Infeld-dilaton theory in the presence of a negative cosmological constant. Note that in AdS case with $\eta = -1$, (4.5) is always meaningful. Thus when this bound is satisfied, (4.4) has positive solution for $r_{\rm H}$, i.e., a extremal black hole indeed exists.[4] This bound indeed relaxes in the limit $b \to \infty$. We focus on the case with negative cosmological constant in the following discussion.

In the presence of a negative cosmological constant $\Lambda = -3/\ell^2$, the exact expression of r_H is complicated due to (4.4), which is a biquadratic algebraic equation for r_H^2. In the limit $\ell \to \infty$, the result is simply $r_{\rm H} = \left(4Q_{\rm e}^2Q_{\rm m}^2 - Q_{\rm e}^2/b^2\right)^{\frac{1}{4}}$, which is the horizon radius in asymptotically flat spacetiem as given in [37, 38]. In the limit $b \to \infty$, Born-Infeld theory reduces to Maxwell theory, and ϕ_0, $\alpha_{\rm H}$ and $r_{\rm H}$ approach values in Einstein-Maxwell-Dilaton theory in asymptotically AdS spacetime [11, 54]. For example (4.4) can be solved perturbatively:

$$\begin{aligned} r_{\rm H}^2 = &\frac{1}{6}\left(-\ell^2 + \ell\sqrt{\ell^2 + 24Q_{\rm e}Q_{\rm m}}\right) \\ &- \frac{1}{b^2}\frac{54Q_{\rm e}^3Q_{\rm m}}{\left(-\ell^2 + \ell\sqrt{\ell^2+24Q_{\rm e}Q_{\rm m}}\right)\left(\ell^2 + 30Q_{\rm e}Q_{\rm m} - \ell\sqrt{\ell^2+24Q_{\rm e}Q_{\rm m}}\right)} + \cdots\,,\end{aligned} \tag{4.6}$$

the first term in the second line of the above expression is the leading Born-Infeld correction to the horizon radius $r_{\rm H}^2$ of the extremal Reissner-Nordström-AdS black hole in the large b limit. Also, it is well-known that the extremal RN-AdS black hole contains AdS_2 as part of its near-horizon geometry. From (4.5) we get

$$\alpha_{\rm H}^2 = 1 + \frac{6}{\ell^2}r_{\rm H}^2 - \frac{Q_{\rm e}^2}{b^2 r_{\rm H}^2}\left(1 + \frac{3}{\ell^2}r_{\rm H}^2\right) + \cdots\,, \tag{4.7}$$

[4] Note that this result is consistent with the proposal in [44] about the non-existence of the extremal limit for purely electrically charged black holes in Einstein-Born-Infeld theories.

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again the third term in (4.7) can be understood as the lowest order Born-Infeld corrections to $\alpha_{\rm H}^2$, which is related to the size of AdS_2.

First order results. At first order, we can write

$$\begin{aligned}\alpha^2(r) &= \alpha_{\rm H}^2 x^2 + \delta\alpha^2 \equiv \alpha_{\rm H}^2 x^2 (1 + a_{1,0} x^\lambda + a_{0,1} x) \ , \\ \beta(r) &= r_{\rm H} + \delta\beta \equiv r_{\rm H}(1 + b_{1,0} x^\lambda + b_{0,1} x) \ , \\ \phi(r) &= \phi_0 + \delta\phi \equiv \phi_0 + \phi_{1,0} x^\lambda + \phi_{0,1} x \ .\end{aligned} \tag{4.8}$$

Substituting (4.8) into the equations of motion (3.8)–(3.11) and keeping $\delta(\alpha^2)$, $\delta\beta$ and $\delta\phi$ as small parameters in perturbation, we get linearized equations in terms of $\delta(\alpha^2)$, $\delta\beta$ and $\delta\phi$. Thus the undetermined coefficients $a_{1,0}$, etc., and λ can be read out from the expansions. From the equation of motion for the scalar we get:

$$\delta\phi = \phi_{1,0}\left(\frac{r}{r_{\rm H}} - 1\right)^\lambda + \phi_{0,1}\left(\frac{r}{r_{\rm H}} - 1\right) \ , \tag{4.9}$$

where $\phi_{1,0}$ is an undertermined constant, and

$$\phi_{0,1} = \frac{\gamma\left(\alpha_{\rm H}^2 - 3r_{\rm H}^2/\ell^2\right)\left(1 - \alpha_{\rm H}^2 + 6r_{\rm H}^2/\ell^2\right)}{\left(1 + 3r_{\rm H}^2/\ell^2\right)\left((\gamma^2 - 1)\alpha_{\rm H}^2 - \gamma^2 3r_{\rm H}^2/\ell^2\right)} \ , \tag{4.10}$$

note that in the limit $\ell \to \infty$, the above result indeed reduces to $\phi_{0,1} = \gamma(1 - \alpha_{\rm H}^2)/(\gamma^2 - 1)$, which is the case in asymptotically flat spacetime [37, 38]. λ can also be determined as:

$$\lambda = \frac{1}{2}\left(-1 + \sqrt{1 + \frac{4B^2}{r_{\rm H}^2 \alpha_{\rm H}^2}}\right) \ , \tag{4.11}$$

with $B^2 \equiv \frac{1}{2}\frac{\partial^2 V_{\rm eff}}{\partial\phi^2}|_{\phi_0, r_{\rm H}}$. Substituting (4.4) and (4.5) into (4.11) we get

$$\lambda = \frac{1}{2}\left(-1 + \sqrt{1 + 8\gamma^2\left(1 - 3r_{\rm H}^2/\ell^2\right)}\right) \ , \tag{4.12}$$

this result reduces to the asymptotically flat case as $\ell \to \infty$, thus $\frac{B^2}{\alpha_{\rm H}^2 r_{\rm H}^2} = 2\gamma^2$ and $\lambda = \frac{1}{2}\left(-1 + \sqrt{1 + 8\gamma^2}\right)$ as in [37, 38].

As discussed in [37, 38], in comparison to the Einstein-Maxwell theory, here the metric get corrections at the first order perturbation theory in x-expansion. Thus to this order:

$$\begin{aligned}\delta\alpha^2 &= \alpha_{\rm H}^2 a_{1,0}\left(\frac{r}{r_{\rm H}} - 1\right)^{\lambda+2} + \alpha_{\rm H}^2 a_{0,1}\left(\frac{r}{r_{\rm H}} - 1\right)^3 \ , \\ \delta\beta &= r_{\rm H}\left(\frac{r}{r_{\rm H}} - 1\right) \ ,\end{aligned} \tag{4.13}$$

where

$$\begin{aligned}a_{1,0} &= \frac{4\gamma\left(1 - \alpha_{\rm H}^2 + 6r_{\rm H}^2/\ell^2\right)\left(\alpha_{\rm H}^2 - 3r_{\rm H}^2/\ell^2\right)}{(\lambda+1)(\lambda+2)\alpha_{\rm H}^2\left(1 + 3r_{\rm H}^2/\ell^2\right)}\phi_{1,0} \ , \\ a_{0,1} &= \frac{2(\alpha_{\rm H}^2 - 3r_{\rm H}^2/\ell^2)}{2(1 + 3r_{\rm H}^2/\ell^2)^2}\left\{1 - \alpha_{\rm H}^2 + \alpha_{\rm H}^4 + \frac{3r_{\rm H}^2}{\ell^2}\left(3 + \alpha_{\rm H}^2 + \frac{3r_{\rm H}^2}{\ell^2}\right)\right. \\ &\qquad \left. + \gamma^2\left(1 - \alpha_{\rm H}^2 + \frac{6r_{\rm H}^2}{\ell^2}\right)^2 \phi_{0,1}\right\} + \frac{2r_{\rm H}^2}{\ell^2} - \frac{4}{3} \ .\end{aligned} \tag{4.14}$$

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Note that from (4.12), the assumption $\lambda \geq 1$ demands that $\gamma^2(1 - 3r_{\rm H}^2/\ell^2) \geq 1$, this can only be satisfied when $\gamma > 1$. When this condition is satisfied, this corrections (4.14) vanishes at the horizon faster than $(r/r_{\rm H} - 1)^2$, thus to this order in x-expansion, the solution continues to be a double horizon black hole with vanishing surface gravity.

Higher order results. At the second order in x-expansion, the value of scalar field we found at first order (4.9)–(4.10) plays the role of a source, which results in corrections to the metric and the scalar field itself. This can be calculated in a similar way as the first order analysis.

In our perturbation analysis, we solve the equations of motion of our system order by order in the x-expansion. As argued in [29, 37, 38], we have seen that to the first order, there is one parameter $\phi_{1,0}$ cannot be determined by the equations of motion themselves. Let us denote the value of $\phi_{1,0}$ as K. We thus find $a_{1,0}$ and $b_{1,0}$ as functions of K. At any order $n \geq 2$, we can substitute the resulting values of $(a_{m,l}, b_{m,l}, \phi_{m,l})$, for all $m + l \leq n$ from the previous orders. Thus (3.8), (3.10), (3.11) of order n and (3.9) of order $(n-1)$ give

$$a_{n,l} = a_{n,l}(K), \qquad b_{n,l} = b_{n,l}(K), \qquad \phi_{n,l} = \phi_{n,l}(K)\ , \tag{4.15}$$

i.e., as polynomials of order n in terms of K. K remains a free parameter to all orders in the x-expansions. From the Frobenius expansion (4.1), all the coefficients are functions of the single parameter K, thus the full black hole solutions, especially the asymptotic values of $\alpha(r)$, $\beta(r)$ and $\phi(r)$ are dependent on the parameter K. After changing bases from $\left(\frac{r}{r_{\rm H}} - 1\right)$ to $\left(1 - \frac{r_{\rm H}}{r}\right)$, it can be shown that $\alpha(\infty)$, $\beta(\infty)$ and $\phi(\infty)$ are free to take different values given by different choices of K. The convergence of this series should be addressed in detail, but it should be convergent when $|K|$ is small enough. The fact that the dilaton $\phi(r)$ can take arbitrary value at asymptotic infinity $\phi(\infty)$ while its value at the horizon is fixed to be ϕ_0 as given in (4.3) shows the presence of attractor mechanism.

5. Summary and discussion

In this note we studied attractor mechanism in Einstein-Born-Infeld theory coupled to scalars and with dilaton-like gauge couplings, in the presence of a cosmological constant in the action. We derived the equations of motion of the system, and looked for possible extremal black hole solutions with proper boundary conditions using a perturbative method.

We focused on the case of asymptotic AdS black holes, which are more interesting due to the AdS/CFT correspondence. We discussed the existence of the extremal black hole solutions, calculated the double-zero horizon radius and the attractor value of the dilaton. It is shown that there are different extremal black hole solutions characterized by different values of the scalars at asymptotic infinity, while the scalar fields are indeed attracted to certain fixed values at the horizon. This result generalizes the analysis in [29, 37, 38] and implies the presence of attractor mechanism in the theory.

One can also study the case in asymptotic dS spacetime, though the analysis of existence of such extremal black holes with desired boundary condition is more complicated.

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Acknowledgments

We would like to thank Miao Li, Da-Wei Pang, Tower Wang and Zhao-Long Wang for useful discussions and kind help. We are grateful to Miao Li for a careful reading of the manuscript and valuable suggestions. This work was supported by grants from CNSF.

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